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A general analytical model for hydraulic computation of open channels with steady state uniform flow

Ioan Sarbu *, Anton Iosif

Department of Building Services Engineering, Polytechnic University of Timisoara, Piata Victoriei 2A, 300006 Timisoara, Romania

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ABSTRACT

In this paper, a generalised mathematical model is developed, which allows for definition in a unit manner an analytical solution of the complex problems of hydraulic computation for the steady state flow open channels. These problems include the design and operational examination for the channels with linear cross-section (trapezoidal, rectangular, and triangular), the channels with curved cross-section (semi-circular, parabolic, and semi-elliptic), as well as the channels with flat sides and a cylindrical bottom. The conditions on hydraulically optimal sections for these channel-types are determined by the authors. The proposed model is programmed easily on microcomputers. Therefore, two computer programs are performed in the FORTRAN programming language for PC-compatible systems, with a view to increase the accuracy and computational efficiency. The advantages to use the proposed programs are explained from two numerical applications for different constructive variants employed in practical engineering.

1. Introduction

Among all the problems that water researchers and hydraulic engineers have faced, water conveyance is considered to be one of not only the inevitable but also the expensive problems. In fact, water conveyance is a means to meet some of the needs of human society, such as irrigation, municipal and flood control. Artificial open channels have been widely used for this purpose because they can be constructed on different topographies and soil conditions and also prevent the wasting of water.

Selection of the cross-sectional shape of open channels is one of the primary issues of hydraulic, constructive, economic and operational orders.

The comparison performed by Mateescu (1963) between different isoperimetric outlines of open channels, each the optimum of its type, shows that the most hydraulically efficient shape is the semi-circle, followed closely by the parabola and the isosceles trapezium with side angles of 60°. For the conveyance of a given water flow rate, the cross-sectional area of a parabolic channel is lower than the hydraulically equivalent trapezoidal channel, as the generic parabola degree is higher.

The comparative study developed by Blidaru et al. (1996) between the parabolic and semi-elliptic channels concludes that the semi-elliptic cross-section has better hydraulic characteristics than the parabolic section because the conveyance of the same liquid flow rate has a smaller area.

As a result, it is a known that for open channels, modern techniques use both linear cross-sections (trapezoidal, rectangular, and triangular) as well as curved cross-sections (parabolic, semi-elliptic, and semi-circular) (Chaudhry, 1993). In addition, taking into account the construction deficiencies of trapezoidal channels, the industrialisation necessity of the hydro-amelioration and hydro-urban systems, and the hydraulic advantages (Chow, 1973; Chaudhry, 1993; Subramanya, 1998), compound cross-section channels with flat sides and cylindrical bottom were adopted in engineering practice. Design and operational validation of such cross-sections was and still is an active area of research.

Chow (1973) and French (1994) have published the most hydraulically efficient section relations. Their objective function was minimisation of the flow area while the Manning's equation was the constraint. Swamee and Bhatia (1972) developed optimal design curves for trapezoidal, rounded bottom and rounded corner sections. The round-bottom triangle is an approximation of the parabola. Loganathan (1991) studied optimality conditions for a parabolic channel section. Monadjemi (1994) showed that the same optimal section variables can

* Corresponding Author.
Email Address: ioan.sarbu@upt.ro (I. Sarbu)
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be achieved by minimisation of either flow rate or wetted perimeter. Froehlich (1994) recommended simple relations optimum section variables of trapezoidal sections in terms of discharge. Although more similar researches were conducted utilising new optimisation techniques (Turan and Yurdusev, 2011; Kaveh et al., 2012), the proposed models were not sufficiently precise in comparison with the benchmark solutions.

In this paper, a general analytical model is developed that solves in a unitary manner the complex problems of hydraulic computation for open channels with steady state uniform flow and can be easily programmed and implemented on microcomputers. Starting with some theoretical aspects of optimal hydraulic computation of open-channels, the conditions of hydraulically optimal sections are determined for channels with simple curve sections (circular, parabolic, and semi-elliptic) as well as for compound section channels with flat sides and a cylindrical bottom. To increase the precision and efficiency of the computations, two computer programs are developed in FORTRAN programming language for personal computer (PC)-compatible systems. The advantages to using the proposed programs are highlighted by two examples of comparative design for different channel cross-sections used in practical engineering.

2. Simple channels

2.1. Geometrical elements of the cross-section

Geometrical elements are properties of a channel section that can be defined entirely by the geometry of the section and the depth of flow. These elements are very important and are used extensively in flow computations.

For simple regular channel sections the geometric elements can be expressed mathematically in terms of the depth of flow and other dimensions of the section. For complicated sections, however, no simple formula can be written to express these elements, but curves representing the relation between these elements and the depth of flow can be prepared for use in hydraulic computations.

1. *Trapezoidal isosceles channel* with the side slope 1/m=tgθ (Fig. 1) has the following geometrical elements:

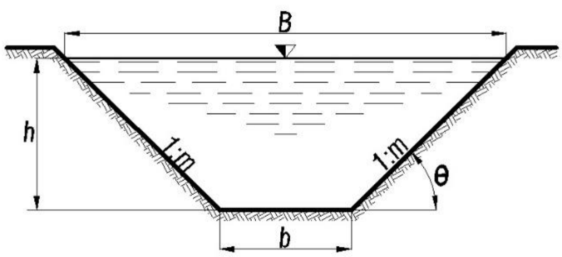

Fig. 1: Trapezoidal channel

- Surface width (B), the width of the channel section at water free-surface:

$$B = b + 2mh \tag{1}$$

where h is the water depth, i.e. the vertical distance from the lowest point of the channel section to the free-surface.

- Area (A), the cross-sectional area of flow, normal to the direction of flow:

$$A = (b + mh)h \tag{2}$$

- Wetted perimeter (P_u), the length of the wetted surface measured normal to the direction of flow:

$$P_u = b + 2h\sqrt{1+m^2} \tag{3}$$

- Hydraulic radius (R) – the ratio of area to wetted perimeter:

$$R = \frac{A}{P_u} = \frac{(2\sqrt{1+m^2} - m)h^2}{b + 2h\sqrt{1+m^2}} \tag{4}$$

2. *Parabolic channel* in generalised form in Fig. 2 is represented by a parabola of α degree defined by equation:

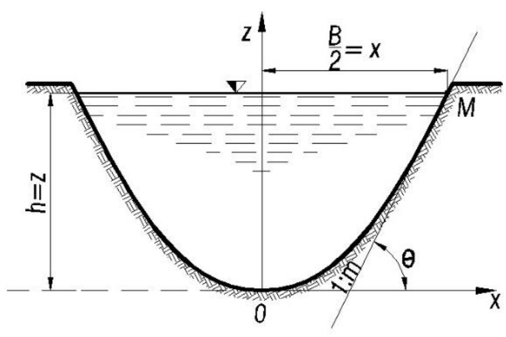

Fig. 2: Parabolic channel

$$z = px^{\alpha} \tag{5}$$

where p is a dimensionless parameter. The following geometrical elements are obtained:

- Section width at water free-surface:

$$B = 2x \tag{6}$$

- Water depth:

$$h = z \tag{7}$$

- Geometric tangent slope in parabola intersection point M with water free-surface plane (side slope):

$$\frac{1}{m} = \text{tg}\theta = \frac{dz}{dx} \tag{8}$$

Differentiating Eq. 5 and stating point M is on the parabola, from Eq. 8, the following results:

$$m = \frac{1}{\alpha p^{1/\alpha}} h^{\frac{1-\alpha}{\alpha}} = \frac{2^{\alpha-1}}{\alpha p} B^{1-\alpha} \qquad (9)$$

- Cross-sectional area:

$$A = 2\int_0^h x(z)dz = 2\frac{\alpha^2 m}{1+\alpha} h^2 \qquad (10)$$

- Wetted perimeter:

$$P_u = 2\int_0^h \sqrt{1+(\frac{dx}{dz})^2 dz} = 2\int_0^h \sqrt{1+m^2}dz \qquad (11)$$

In the case that the generic parabola is of random form, P_u can be expressed as:

$$P_u = \lambda\sqrt{A} \qquad (12)$$

where, the parameter $\lambda=\lambda(\alpha,m)$ can be calculated with a satisfactory approximation using the following relations (Swamee, 1995; Sarbu and Kalmar, 2000):

a) For $m \geq 1$ and $\alpha>1$:

$$\lambda = \sqrt{2(1+\alpha)m}\ \left[1+\frac{1}{2(2\alpha-1)m^2}-\frac{1}{8(4\alpha-3)m^4}\right] \qquad (13)$$

b) For $m<1$ and $\alpha=2$:

$$\lambda = \sqrt{2(1+\alpha)}\ m^{\frac{1+\alpha}{2(\alpha-1)}}\left[\frac{8\alpha^2-8.25\alpha+1.625}{(4\alpha-3)(2\alpha-1)}+\frac{1}{\alpha}\left(m^{\frac{\alpha}{1-\alpha}}-1\right)+\frac{1}{8(3\alpha-4)}\left(m^{\frac{4-3\alpha}{1-\alpha}}-1\right)\right] \qquad (14)$$

c) For $m<1$ and $\alpha<1$ ($\alpha\neq2$):

$$\lambda = \sqrt{2(1+\alpha)}\ m^{\frac{1+\alpha}{2(\alpha-1)}}\left[\frac{8\alpha^2-8.25\alpha+1.625}{(4\alpha-3)(2\alpha-1)}+\frac{1}{\alpha}\left(m^{\frac{\alpha}{1-\alpha}}-1\right)-\frac{1}{2(\alpha-2)}\left(m^{\frac{2-\alpha}{1-\alpha}}-1\right)+\frac{1}{8(3\alpha-4)}\left(m^{\frac{4-3\alpha}{1-\alpha}}-1\right)\right] \qquad (15)$$

- Hydraulic radius:

$$R = \frac{A}{P_u} = \sqrt{2}\frac{\alpha}{\lambda}\sqrt{\frac{m}{1+\alpha}}\ h \qquad (16)$$

3. *Semi-elliptic channel* in Fig. 3 is defined by the general ellipse with the following equation:

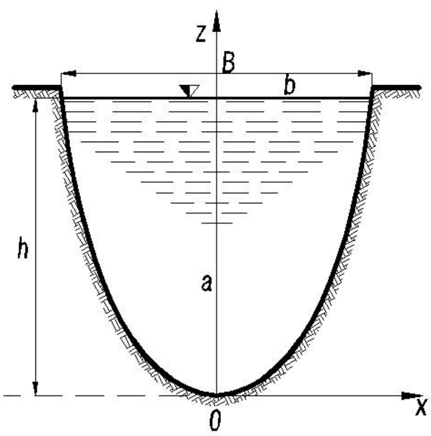

Fig. 3: Semi-elliptic channel

$$\frac{(z-a)^2}{a^2}+\frac{x^2}{b^2}-1=0 \qquad (17)$$

where, a is the major semi-axis; b is the small semi-axis. The following expressions for cross-section elements are obtained:

- Section width at water free-surface:

$$B = 2b \qquad (18)$$

- Water depth:

$$h = a \qquad (19)$$

- Cross-sectional area:

$$A = \frac{\pi}{2}ab = \frac{\pi}{4}Bh \qquad (20)$$

- Wetted perimeter (iMatematica, 2016):

$$P_u = \pi(\frac{3}{2}\frac{a+b}{2}-\frac{1}{2}\sqrt{ab} \qquad (21)$$

- Hydraulic radius:

$$R = \frac{A}{P_u}\ \frac{B}{1.5\ (\frac{B}{h}+2)-\sqrt{2\frac{B}{h}}} \qquad (22)$$

The following dimensionless parameters $\varphi=B/h$, $\psi=A/h^2$, and $f=P_u/h$ are defined as:

for trapezoidal section:

$$\beta = \frac{b}{h} \qquad (23)$$
$$\varphi = \beta+2m \qquad (24)$$
$$\psi = \beta+m \qquad (25)$$
$$f = \beta+2\ \sqrt{1+m^2} \qquad (26)$$

for parabolic section:

$$\varphi = 2\alpha m \qquad (27)$$
$$\psi = 2\ \frac{\alpha^2 m}{1+\alpha} \qquad (28)$$
$$f = \sqrt{2}\ \lambda\ \alpha\ \sqrt{\frac{m}{1+\alpha}} \qquad (29)$$

for semi-elliptic section:

$$\varphi = \frac{B}{h} \qquad (30)$$
$$\psi = \frac{\pi}{4}\ \varphi \qquad (31)$$
$$f = \frac{\pi}{4}[1.5\ (\varphi+2)-\sqrt{2\varphi}], \qquad (32)$$

Each dimensional geometrical element can be expressed depending on one sole dimensional element and two other dimensionless parameters, according to Table 1.

For m=0 and β=0, in Eqs. 23-26, the particular case of a rectangular channel and a triangular channel, respectively, is obtained. Substituting φ=2 in Eqs. 30-32 correspond to the case of a semi-circular channel.

2.2. Hydraulically optimal section

The hydraulically optimal cross-section (Arsenie and Arsenie, 1981) of an open channel is a section that for the same area A, the same bottom slope i and the same roughness coefficient n, conveys the maximum discharge Q. Maximum discharge is obtained when the hydraulic radius is maximal or when the wetted perimeter is minimal.

Table 1: Geometrical element expression of simple cross-sections

Elements	A function of:				
	h	B	A	P_u	R
h	h	$\frac{1}{\varphi}B$	$\frac{1}{\psi^{0,5}}\sqrt{A}$	$\frac{1}{f}P_u$	$\frac{f}{\psi}R$
B	φh	B	$\frac{\varphi}{\psi^{0,5}}\sqrt{A}$	$\frac{\varphi}{f}P_u$	$\frac{\varphi f}{\psi}R$
A	ψh^2	$\frac{\psi}{\varphi^2}B^2$	A	$\frac{\psi}{f^2}P_u^2$	$\frac{f^2}{\psi}R^2$
P_u	fh	$\frac{1}{\varphi}B$	$\frac{1}{\psi^{0,5}}\sqrt{A}$	P_u	$\frac{f^2}{\psi}R$
R	$\frac{\psi}{f}h$	$\frac{\psi}{\varphi f}B$	$\frac{\psi^{0,5}}{f}\sqrt{A}$	$\frac{\psi}{f^2}P_u$	R

For *the trapezoidal channel*, the value $b=A/h-mh$ is substituted in the expression of wetted perimeter P_u and writing the minimum condition:

$$\frac{dP_u}{dh} = -\frac{A}{h^2} - m + 2\sqrt{1+m^2} = 0, \qquad (33)$$

is obtained:

$$b = 2h\ tg\frac{\theta}{2} \qquad (34)$$

Eq. 34 shows that the hydraulically optimal cross-section, for a given slope, corresponds to:

$$\beta_o = \frac{b}{h} = 2\ tg\frac{\theta}{2} \qquad (35)$$

If the minimal perimeter condition is imposed, then θ=60°.

For *the parabolic channel*, the hydraulically optimal cross-section condition is obtained by determining the minimum of function (12), considering A=const., which is reduced to calculate the minimum of function $\lambda(\alpha, m)$. For this purpose, the values $\lambda(\alpha, m)$ were calculated using Eqs. 13-15 for 80 values of α and 340 values of m using a computer program.

The minimum value $\lambda(\alpha, m)$ for a given degree α of the generic parabola corresponds to the reverse optimal side slope m_o, of which the values are presented in Table 2 and represent the condition of hydraulically optimal parabolic cross-section.

The minimal value $\lambda(\alpha, m)$ for a given slope $1/m$ also corresponds to α_o being the optimal degree of a generic parabola. The optimal value α_o is listed in Table 3 and represent the hydraulically optimal parabolic cross-section condition for a given side slope.

Table 2: Optimal value m_o for various α for parabolic channels

α	1.5	2.0	2.5	3.0	3.5	4.0	4.5
m_o	0.851	0.521	0.517	0.425	0.356	0.307	0.272
α	5.0	6.0	7.0	8.0	9.0	10.0	–
m_o	0.241	0.198	0.167	0.145	0.128	0.114	–

Table 3: Optimal value α_o for various m for parabolic channels

m	0.2	0.3	0.4	0.50	0.60	0.70	0.80	0.90	1.00
α_o	5.5	4.0	3.1	2.6	2.2	1.9	1.7	1.4	1.3

For *the semi-elliptic channel,* the hydraulically optimal section condition is obtained by considering A=const. and determining the minimum of function (21) rewritten under general form:

$$P_u = \frac{\sqrt{\pi}}{2}\left(1.5\frac{\varphi+2}{\sqrt{\varphi}} - \sqrt{2}\right)\sqrt{A}\,, \qquad (36)$$

The following is obtained:

$$\varphi_o = 2\,, \qquad (37)$$

Condition (37) leads to the limit case when the semi-ellipse tends to a semi-circle.

2.3. Mathematical model

Expressing water velocity V using Chézy's formula (38) and adopting the Pavlovski's formula (39) for the hydraulic resistance coefficient C yields the well-known Q discharge equation (40) (Chow, 1973; Chaudhry, 1993; French, 1994; Subramanya, 1998):

$$V = C\sqrt{Ri} \qquad (38)$$

$$C = \frac{1}{n}R^y \qquad (39)$$

$$Q = VA = \frac{1}{n}AR^{y+0.5}i^{0.5} \qquad (40)$$

where, Q is the volumetric discharge; n is the Manning's roughness coefficient; A is the cross-sectional area; R is the hydraulic radius; i is the channel bottom slope; and y is an exponent in Pavlovski's formula, calculated with the following equation:

$$y = 2.5\sqrt{n} - 0.13 - 0.75\sqrt{R}\left(\sqrt{n} - 0.10\right) \qquad (41)$$

Eq. 41 is valid for R between 0.1 and 0.3 and for n between 0.011 and 0.040.

Substituting the A and R expressions depending on h (Table 1) in Eq. (40), the general equation for hydraulic computation of simple cross-section channels with steady state uniform flow is obtained:

$$Q = \frac{1}{n}\frac{\psi^{y+1.5}}{f^{y+0.5}}h^{y+2.5}i^{0.5} \qquad (42)$$

If the water velocity V obtained after design is not enclosed between admissible limits indicated in the literature (Chow, 1973), such as minimum permissible velocity V_m, that avoids the deposition of sediment and suspensions, and maximum permissible velocity V_M, that are safe against erosion, the channel bottom slope i is increased or decreased according to Eq. 43 obtained from the Chézy's formula (38):

$$i = \frac{n^2 V^2_{m(M)}}{R^{2y+1}}\,, \qquad (43)$$

and the water depth h is determined with the following equation:

$$h = \frac{1}{\psi^{0.5}}\sqrt{\frac{Q}{V_{\mathrm{m(M)}}}} \quad (44)$$

The design of a channel with a given shape requires determination of the variables h, B and V when the elements Q, i, n and parameters α and p or m (parabolic section), φ (semi-elliptic or semi-circular section), m and β (trapezoidal section), m (triangular section), or β (rectangular section) are known. This can be easily performed applying the iteration method. In the case of the operation checking problem, the geometrical elements B, h, i, n, m and α are given and the hydraulic elements Q and V are computed.

Taking into account the calculation formulas presented, an algorithm to solve the outstanding problems of the hydraulic computation of simple cross-section (linear and curve) open channels with steady state uniform flow was developed. On the basis of this algorithm the computer program CANDES1 (Sarbu and Kalmar, 2000) was elaborated in FORTRAN programming language for PC microsystems.

3. Compound channels

3.1. Geometrical elements of the channels with flat sides and a cylindrical bottom

The channel with flat sides and a cylindrical bottom (round-bottomed triangle) is a form usually created by excavation with shovels. Taking into account the notations in Fig. 4 and the tangency condition of trapezium sides to the circle arch, the cross-section elements can be deduced:

$$m = \mathrm{ctg}\theta \quad (45)$$
$$b = 2r\sin\theta = f_1(m)r \quad (46)$$
$$h = 2r\sin^2\frac{\theta}{2} = f_2(m)r \quad (47)$$
$$H - h = \alpha r \quad (48)$$

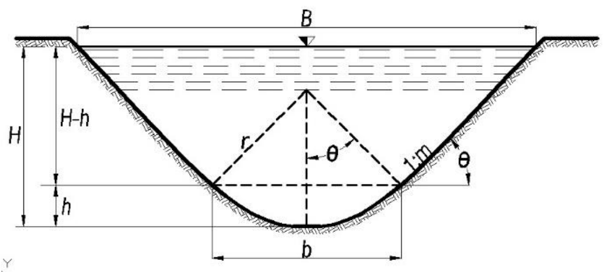

Fig. 4: Channel with flat sides and a cylindrical bottom

$$H = \frac{\alpha + f_2(m)}{f_2(m)}h = [\alpha + f_2(m)]\,r \quad (49)$$
$$B = b + 2m(H-h) = [2m\alpha + f_1(m)]\,r \quad (50)$$

in which:

$$f_1(m) = \frac{4}{m\prime} \quad (51)$$
$$m' = 2\sqrt{1+m^2} \quad (52)$$
$$f_2(m) = 1 - \frac{2m}{m\prime} \quad (53)$$
$$\alpha = \frac{H}{r} - f_2(m) = f_2(m)\left(\frac{H}{h} - 1\right) \quad (54)$$

where, θ is the side angle; m is the reciprocal of the side slope; r is the circle arch radius; H is the water depth; B is the section width at the water free-surface.

Using Eqs. 49 and 50, the following is obtained:

$$\beta = \frac{B}{H} = \frac{2m\alpha + f_1(m)}{\alpha + f_2(m)} \quad (55)$$
$$\alpha = \frac{f_1(m) - f_2(m)\,\beta}{\beta - 2m} \quad (56)$$

According to Eq. 54, if h=H (circular channel) and h=0 (trapezoidal channel), then α has the value 0 and ∞, respectively.

Introducing restrictions α=0 and α=∞ in Eq. 55, the problem's compatibility condition is obtained:

$$2m < \beta < \frac{4}{m\prime - 2m} \quad (57)$$

Eq. 57 expresses the domain in which the link between the geometric and hydraulic characteristics can be analysed.

For the wetted perimeter P_u, area A and hydraulic radius R, the following expressions are derived:

$$P_u = 2\sqrt{1+m^2}(H-h) + 2\,\theta\, r = [m'\alpha + 2f_3(m)]\,r \quad (58)$$
$$A = [b + m(H-h)]\,(H-h) + \frac{2\,\theta - \sin 2\,\theta}{2}r^2 = [m\alpha^2 + f_1(m)\alpha + f_4(m)]\,r^2 \quad (59)$$
$$R = \frac{A}{P_u} = \frac{m\alpha^2 + f_1(m)\alpha + f_4(m)}{m\prime\alpha + 2f_3(m)}\;r \quad (60)$$

in which:

$$f_3(m) = \mathrm{arctg}\frac{1}{m} \quad (61)$$
$$f_4(m) = f_3(m) - \frac{2m}{m\prime} \quad (62)$$

Using the following notations:

$$\varphi = H/r = \alpha + f_2(m) \quad (63)$$
$$\tau = B/r = 2m\alpha + f_1(m) \quad (64)$$
$$\psi = \sqrt{A}/r = \sqrt{m\alpha^2 + f_1(m)\alpha + f_4(m)} \quad (65)$$
$$f = P_u/r = m'\alpha + 2f_3(m) \quad (66)$$

every dimensional geometric element is a function of only one dimensional element and two other dimensionless parameters m and α, according to Table 4.

3.2. Mathematical model

Substituting the expression for geometrical elements A and R depending on H (Table 4) in the discharge Eq. 40, the general formula is obtained for the hydraulic computation of channels with flat sides and a cylindrical bottom in steady state uniform flow:

$$Q = \frac{1}{n}\frac{\psi^{2y+3}}{\varphi^{y+2.5}f^{y+0.5}}\,H^{y+2.5}i^{0.5} \quad (67)$$

The expression of radius r obtained in Eq. 59 is substituted in Eq. 58 to yield the following notation:

$$\lambda = \frac{f}{\psi} = \frac{m\prime\alpha+2f_3(m)}{\sqrt{m\alpha^2+f_1(m)\alpha+f_4(m)}} \quad (68)$$

Table 4: Geometrical element expression of the cross-section

Elem.	A function of: r	H	B	A	P_u	R
r	r	$\frac{1}{\varphi}H$	$\frac{1}{\tau}B$	$\frac{1}{\psi^2}\sqrt{A}$	$\frac{1}{f}P_u$	$\frac{f}{\psi^2}R$
H	$\varphi\, r$	H	$\frac{\varphi}{\tau}B$	$\frac{\varphi}{\psi}\sqrt{A}$	$\frac{\varphi}{f}P_u$	$\frac{\varphi\, f}{\psi^2}R$
B	$\tau\, r$	$\frac{\tau}{\varphi}H$	B	$\frac{\tau}{\psi}\sqrt{A}$	$\frac{\tau}{f}P_u$	$\frac{\tau\, f}{\psi^2}R$
A	$\psi^2 r^2$	$\frac{\psi^2}{\varphi^2}H^2$	$\frac{\psi^2}{\tau^2}B^2$	A	$\frac{\psi^2}{f^2}P_u^2$	$\frac{f^2}{\psi^2}R^2$
P_u	$f\, r$	$\frac{\varphi}{f}H$	$\frac{1}{\tau}B$	$\frac{1}{\psi}\sqrt{A}$	P_u	$\frac{f^2}{\psi^2}R$
R	$\frac{\psi^2}{f}r$	$\frac{\psi^2}{\varphi\, f}H$	$\frac{\psi^2}{\tau\, f}B$	$\frac{\psi^{0,5}}{f}\sqrt{A}$	$\frac{\psi^2}{f^2}P_u$	R

Thus, expression (58) of the wetted perimeter becomes:

$$P_u = \lambda\sqrt{A} \quad (69)$$

In the case of a hydraulically optimal section that corresponds to the minimal wetted perimeter for the same A, n and i values, the dimensionless parameter λ becomes minimal and $\alpha=\alpha_o$ is obtained by determination of the (68) function minimum from condition $\partial\lambda/\partial\alpha=0$:

$$\alpha_o = \frac{f_1(m)\, f_3(m)-m\prime f_4(m)}{\frac{1}{2}m\prime f_1(m)-2mf_3(m)} = \frac{2m}{m\prime} = \frac{m}{\sqrt{1+m^2}} \quad (70)$$

Substituting Eq. 70 in Eq. 55 results in the condition of the hydraulically optimal section for a given side slope:

$$\beta_o = m' = 2\sqrt{1+m^2} \quad (71)$$

The values of optimal relative width β_o determined in Eq. 71 for different values m are presented in Table 5.

Table 5: Values of optimal relative width β_o

m	0	0.25	0.50	0.75	1.00	1.25
β_o	2.00	2.00	2.24	2.50	2.83	3.20
m	1.50	2.00	2.50	3.00	4.00	–
β_o	3.61	4.47	5.39	6.32	8.25	–

If the resulting water velocity V after design process is not between admissible limits V_m and V_M, then the channel bottom slope i is increased or decreased according to Eq. 43, and the water depth H is determined by the equation:

$$H = \frac{\varphi}{\psi}\sqrt{\frac{Q}{V_{m(M)}}} \quad (72)$$

The mathematical model described was implemented in the computer program CANDES2 (Sarbu and Kalmar, 2000) written in FORTRAN programming language for PC microsystems.

4. Numerical applications

To enhance the advantage of the CANDES1 program, the design of a channel was performed considering different cross-sections (trapezoidal, rectangular, triangular, parabolic, semi-elliptic, and semi-circular) and using the following data: Q=5 m^3/s, i=0.0006, n=0.014, V_m=0.7 m/s and V_M=4 m/s. The numerical results of the computations are summarised in Table 6. The design of a hydraulically optimal section of a channel with flat sides and a cylindrical bottom was obtained using CANDES2 program for Q=10.0 m^3/s, i=0.0006, n=0.0225, m=1, V_m=0.50 m/s, V_M=1.50 m/s and ε=0.001 m. The following numerical results are obtained: β_o=2.828, r=2.373 m, h=0.695 m, b=3.355 m, H=2.373 m, B=6.711 m, V=1.126 m/s and i=0.0006.

Table 6: Characteristic elements provided by the CANDES1 program

No.	Computed elements	Cross-section shapes: Trapezoidal	Rectangular	Triangular	Parabolic	Semi-elliptic	Semi-circular
		α=0, β=0.83, m = 2	α = 0, β = 2, m = 0	α = 0, β=0, m = 1	α = 5, β=0, m = 0.241	α = 0, β = 0, φ = 1.6	α = 0, β = 0, φ = 2
1	h [m]	1.405	1.359	1.922	1.320	1.668	1.488
2	B [m]	3.976	2.718	3.844	3.181	2.668	2.976
3	A [m^2]	3.612	3.694	3.694	3.500	3.493	3.477
4	P_u [m]	5.140	5.436	5.436	4.750	4.728	4.673
5	V [m/s]	1.389	1.359	1.359	1.433	1.436	1.442
6	i [%]	0.060	0.060	0.060	0.060	0.060	0.060
7	b [m]	1.166	2.718	–	–	–	–
8	p [m^{-4}]	–	–	–	0.130	–	–

According to the simplicity and accuracy of the proposed models, the lined channel design using explicit relation probably can be of interest not only practical projects but also for future studies while more channel sections and alternative algorithms may be considered.

5. Conclusion

A unifying computation model is derived for channel cross-sections with simple geometries, as well as a model for compound cross-sections. These models have been coded into computer programs for easy utilisation by practicing engineers.

The proposed mathematical models have a higher degree of generality, which allows for performing a more precise unitary hydraulic computation for the channels with different cross-section shapes and to establish their hydraulically optimal profile. The optimal hydraulic computation of these channels can be typically applied for the precast conveyer troughs with different shapes within hydro-amelioration systems.

The general hydraulic calculation of channels with flat sides and a cylindrical bottom is based on Eq. 67. This calculation is performed in a manner analogous to the hydraulically optimal cross-section calculation developed in this study. The difference is that the most adequate relations between the geometrical elements m, α, H, B, r, A and R are selected.

Some aspects concerning the hydraulic computation for random degree parabolic channels can be applied in the case of symmetrical river beds (Sarbu and Retezan, 1985).

The developed computer programs allow for performing an efficient computation in a manner that is more precise than the traditional methods, when different constructive variants are compared.

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